The reflection and transmission of plane waves from layered media is a standard problem in advanced undergraduate/introductory postgraduate courses. Controlling the reflection over a frequency band is an interesting problem in optimization. This month's contribution addresses a package developed by the authors to address this. The authors have made the code described publicly available, and we thank them for this submission.

A more advanced application of this type of work is in the design of radar absorbers. Some years back, the design of Jauman absorbers was reviewed in the Magazine [1]. This type of absorber consists of resistive sheets sandwiched between dielectric layers.


A Graphical User Interface for Calculation of the Reflection and Transmission Coefficients of a Layered Medium

Veysel Demir and Atef Z. Elsherbeni

Center of Applied Electromagnetic Systems Research (CASER)
Electrical Engineering Department, The University of Mississippi, University, MS 38677 USA
E-mail: vdemir@olemiss.edu, atef@olemiss.edu

Abstract

This paper presents a software package that calculates the reflection and transmission coefficients of a layered medium, and optimizes reflection and transmission in given ranges of parameters. The algorithms to calculate fields in a layered medium due to normally incident plane waves are described, and the use of a software package developed based on these algorithms is demonstrated.

Keywords: Electromagnetic scattering; electromagnetic scattering by nonhomogeneous media; electromagnetic reflection; nonhomogeneous media; optimization methods: graphical user interfaces
1. Introduction

Reflection and transmission of electromagnetic waves by a layered medium is in the scope of undergraduate/introductory graduate-level electromagnetics courses, and has well-known, straightforward solutions [1, 2]. It is easy to develop some algorithms based on these solutions. Based on these algorithms, a software package utilizing a graphical user interface (GUI) was developed, using MATLAB and FORTRAN, in order to calculate the reflection and transmission coefficients of a layered medium, and to optimize reflection and transmission in given ranges of parameters. The main emphasis of this package is to let students learn how the optimization of a problem — commonly taught at the undergraduate level — can lead to useful applications, and that various applications can also be attempted and fostered using the knowledge gained through their education. A graphical interface is then necessary to allow ease of use for applications, without a need for reprogramming or teaching optimization at this early stage of teaching.

The developed program provides an easy-to-use interface to define and visualize the geometry of the problem, and to display the solutions for reflection and transmission coefficients, as well as the field distributions in the medium. Furthermore, the program helps the user define parameters to sweep, and optimizes reflection or transmission for a given target value within the given ranges of swept parameters.

2. Reflection and Transmission by a Layered Medium

Consider a \( z \)-polarized \( x \)-traveling plane wave, \( \vec{E}_0 \), which is normally incident on a layered medium, as shown in Figure 1. Each layer, \( "n" \), is defined by its permittivity \( \varepsilon_i \), permeability \( \mu_i \), electric conductivity \( \sigma_i^e \), magnetic conductivity \( \sigma_i^m \), and thickness \( d_i - d_{i-1} \). Backward- and forward-traveling waves in any layer \( i \) are denoted by \( \vec{A}_i \) and \( \vec{B}_i \), respectively, where expressions for these waves are given by

\[
\vec{A}_i = zA_i e^{jkd_i},
\]

\[
\vec{B}_i = zB_i e^{-jkd_i}.
\]

The total field, \( \vec{E}_i^{\text{total}} \), in layer \( i \) is the sum of the forward- and backward-traveling waves, such that

\[
\vec{E}_i = \vec{A}_i + \vec{B}_i = z \left( A_i e^{jkd_i} + B_i e^{-jkd_i} \right).
\]

If the medium consists of \( N \) layers that are surrounded by semi-infinite dielectric spaces on both sides, the reflection and transmission coefficients are expressed as

\[
\Gamma = \frac{A_0}{E_0},
\]

\[
T = \frac{B_{N+1}}{E_0}.
\]

Figure 1. The layout of the layered medium, and the forward- and backward-traveling waves.

In order to find the reflection and transmission coefficients, the forward- and backward-traveling wave amplitudes will be calculated. These waves should satisfy the continuity of the tangential components of the total electric and magnetic fields on the boundaries. The electric-field continuity equation can be written as

\[
A_{i+1} e^{jkd_i} + B_{i+1} e^{-jkd_i} = A_{i+1} e^{jkd_{i+1}} + B_{i+1} e^{-jkd_{i+1}},
\]

whereas the magnetic-field continuity equation can be written as

\[
\frac{\varepsilon_i}{\mu_i} \left( A_{i} e^{jkd_{i}} - B_{i} e^{-jkd_{i}} \right) = \frac{\varepsilon_{i+1}}{\mu_{i+1}} \left( A_{i+1} e^{jkd_{i+1}} - B_{i+1} e^{-jkd_{i+1}} \right).
\]

The term \( k_i \) is the wavenumber in layer \( i \), and can be written as

\[
k_i = \omega \sqrt{\varepsilon_i \mu_i},
\]

where \( \omega \) is the angular frequency of the waves. The complex permittivity, \( \varepsilon_i' \), and the permeability, \( \mu_i' \), are given by

\[
\varepsilon_i' = \varepsilon_i \left( 1 - j \frac{\sigma_i^e}{\omega} \right),
\]

\[
\mu_i' = \mu_i \left( 1 - j \frac{\sigma_i^m}{\omega} \right).
\]

Equations (4) can be rearranged into the forms

\[
Cxe_i B_i + Cye_i A_i + Cze_i B_{i+1} + Cwe_i A_{i+1} = 0,
\]

\[
Cxe_i B_i + Cye_i A_i + Cze_i B_{i+1} + Cwe_i A_{i+1} = 0.
\]

where

\[
Cxe_i = e^{-jkd_i},
\]

\[
Cye_i = e^{jkd_i},
\]

\[
Cze_i = -e^{-jkd_{i+1}},
\]

\[
Cwe_i = -e^{jkd_{i+1}},
\]

\[
Cy_{i} = \sqrt{\varepsilon_i/\mu_i} e^{jkd_i},
\]

\[ C \hat{h}_i = -\sqrt{\varepsilon_i / \mu_i} e^{-jk_i d_i}, \]
\[ C \hat{h}_i = -\sqrt{\varepsilon_{i+1} / \mu_{i+1}} e^{jk_{i+1} d_i}, \]
\[ C \hat{h}_i = \sqrt{\varepsilon_i / \mu_i} e^{-jk_i d_i}. \]

In addition to Equations (7), we assume that

\[ \hat{E}_0 = \tilde{E}_0, \quad (8a) \]
\[ A_{N+1} = 0. \quad (8b) \]

Using Equations (7) and (8), a matrix equation can be constructed. For the configuration in Figure 1, it can be written as shown in Figure 2, and will be denoted Equation (9). Solution of Equation (9) will give the amplitudes of the forward- and backward-traveling waves in all layers. However, solution of Equation (9) is not computationally efficient, and one can construct another algorithm, denoted as the back-to-front algorithm, which is more efficient.

Let’s assume that \( E_0 \) is not known, and \( B_{N+1} = 1 \). We know that \( A_{N+1} = 0 \), because there is no reflection within the right semi-infinite medium. Therefore, the wave amplitudes in layer \( N \) can be determined, since examination of Equation (7) reveals that if the amplitudes \( A_{i+1} \) and \( B_{i+1} \) in layer \( i+1 \) are known, Equation (7) can be rearranged and solved simultaneously for the amplitudes \( A_i \) and \( B_i \) in layer \( i \), such that

\[
\begin{bmatrix}
    B_i \\
    A_i
\end{bmatrix} = \frac{1}{Cxe_i Cye_i - Cxe_i Cze_i - Cze_i Cwe_i - Cwe_i} \begin{bmatrix}
    Cxe_i & -Cye_i & -Cze_i & -Cwe_i \\
    Cxe_i & Cze_i & Cwe_i & Cxe_i \\
    -Cze_i & Cwe_i & Cxe_i & Cye_i \\
    Cwh_i & Cze_i & Cwe_i & Cxe_i \\
    \vdots & \vdots & \vdots & \vdots
\end{bmatrix} \begin{bmatrix}
    A_{i+1} \\
    B_{i+1}
\end{bmatrix}
\]

(10)

3. Reflection by a Layered Medium with PEC Termination

So far, it has been assumed that the layered medium is surrounded by semi-infinite dielectric spaces on both sides. However, the equations and algorithms demonstrated in the previous section can also be used to calculate the reflection coefficient and the fields in a layered medium terminated by a PEC (perfect electrical conductor) boundary, as well. In this case, the boundary condition for the tangential electric field components needs to be applied, such that

\[ A_N e^{jk_N d_N} + B_N e^{-jk_N d_N} = 0, \quad (11a) \]

which leads to

\[ Cxe_N B_N + Cye_N A_N = 0. \quad (11b) \]

Using Equation (11) together with Equation (7), one can construct a matrix equation similar to Equation (9): this is Equation (12), given in Figure 3. The back-to-front algorithm that is described in the previous section still can be employed to solve this problem more efficiently instead of solving Equation (12). Assuming that the forward-traveling wave in layer \( N \), the layer touching the PEC boundary on a side, has unit amplitude, i.e.,

\[ B_N = 1, \quad (13) \]

and by using Equation (13) in Equation (11), one can find the backward-traveling wave amplitude as

\[
\begin{bmatrix}
    B_0 \\
    A_0 \\
    B_1 \\
    A_1 \\
    B_2 \\
    A_2 \\
    B_3 \\
    A_3 \\
    \vdots \\
    B_N \\
    A_N \\
    B_{N+1} \\
    A_{N+1}
\end{bmatrix} = \begin{bmatrix}
    Cxe_0 & Cye_0 & Cze_0 & Cwe_0 \\
    Cxe_0 & Cye_0 & Cze_0 & Cwe_0 \\
    Cxe_0 & Cye_0 & Cze_0 & Cwe_0 \\
    Cxe_0 & Cye_0 & Cze_0 & Cwe_0 \\
    \vdots & \vdots & \vdots & \vdots \\
    Cxe_N & Cye_N & Cze_N & Cwe_N \\
    Cxe_N & Cye_N & Cze_N & Cwe_N \\
    Cxe_N & Cye_N & Cze_N & Cwe_N \\
    1
\end{bmatrix} \begin{bmatrix}
    E_0 \\
    0 \\
    0 \\
    0 \\
    0 \\
    0 \\
    0 \\
    0 \\
    B_N \\
    0 \\
    0 \\
    0
\end{bmatrix}
\]

Figure 2. The matrix Equation (9) for the amplitudes of the forward- and backward-traveling waves in all layers.
\begin{align}
A_N = \frac{e^{-jk_Nd_N}}{e^{jk_Nd_N}}. 
\end{align}

Once the amplitudes $A_N$ and $B_N$ are calculated, one can continue with Equation (10) until all other amplitudes are determined.

Furthermore, it is worth noting that the transmission-line analogy is another technique that is often employed to solve these kinds of problems.

\section{4. Description of the Program}

The algorithms described in the previous sections have been programmed, and a software package with a graphical user interface has been constructed. Figure 4 shows the main GUI that is used to define the problem. The user can define multiple numbers of layers. Each layer is defined by its relative permittivity, relative permeability, electric conductivity, magnetic conductivity and thickness. The user defines the units of frequency and length on which the problem parameters are based. By default, the layered medium is surrounded by free space on both sides. However, the user can choose a PEC termination on the right boundary. The reflection and transmission coefficients are calculated due to plane waves incident on the left boundary. The user defines the frequencies of the plane waves. On clicking the plot fields button, another window that displays the results pops up, as shown in Figure 6. The reflection and transmission coefficients can be displayed as functions of frequency in this window. The magnitudes and phases can be displayed separately, using the dropdown menu. The magnitude can be displayed using either a linear scale or a dB scale by using the toggle button. Furthermore, the spatial distribution of the fields at a chosen frequency can be displayed. Any displayed plot can be exported to a separate figure by clicking the plot in separate figure button. This helps the user to manipulate the plot for use in any other documents.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure4.png}
\caption{The GUI for defining the parameters of multiple layers of slabs.}
\end{figure}

In the main problem-definition window, the optimization parameters button pops up the optimization parameters window, as shown in Figure 5. All parameters of the layers are listed in this window. The user can choose some of these parameters as sweeping parameters by clicking on the associated check boxes. When a parameter is chosen as a sweeping parameter, the user can define the range of this parameter and the step size for the values that this parameter can take. The user can choose optimization on the reflection coefficient or the transmission coefficient, using the respective radio button. Furthermore, the user can define a target...
where $NF$ is the number of frequencies in the frequency range, and $\Gamma_n$ and $T_n$ are the reflection and transmission coefficients at the frequency defined by the index $n$. $\Gamma_{target}$ and $T_{target}$ are the target values for the reflection and transmission coefficients. The combination of parameters that satisfies the condition in Equation (15) is considered to be optimum. Since sweeping through all possible combinations and calculating the reflection or transmission coefficient for each combination is a time-consuming process, this task is programmed in FORTRAN rather than MATLAB, in order to obtain a fast response.

Once a problem is constructed and solved, it can be saved as a project file. The user thus does not need to redefine a problem that has been worked on before, and hence a re-optimization for a different set of parameters can be started easily at a later time.

### 5. Sample Results

Figure 4 shows a two-layer medium in free space backed by a PEC wall. The material parameters of the layers are displayed in the figure. Since the layers were backed by a PEC wall and the layer media were lossless, the incident fields were going to be reflected without any losses. If it is desired to minimize the reflection from these layers, the materials should include losses. Figure 5 shows the configuration of an optimization problem that was used to find the optimum electrical conductivity values of the layers for minimum reflection. As shown in Figure 5a, the electrical conductivity parameters were enabled as sweeping parameters. After the optimization was performed, optimum values of the electrical conductivities were obtained within the specified ranges of values, as shown in Figure 5b. Figure 6 shows the amplitudes of the reflection coefficients from the layers using these new electrical conductivity values, over a frequency range from 8 to 12 GHz, on a dB scale.

Figure 5a. Sweeping parameters for minimum reflection: Electrical conductivities are enabled as sweeping parameters and their ranges are defined.

Figure 5b. After sweeping has been performed, optimum values of the parameters are determined.

value for optimization. The start optimization button runs executable code that sweeps all combinations of parameters, and finds the combination that gives the closest reflection or transmission to the target value. It returns this combination back to the optimization parameters window. The condition for the optimum value is given by

\[
\min \left( \sum_{n=1}^{NF} |\Gamma_n - \Gamma_{target}|^2 \right), \tag{15a}
\]

or

\[
\min \left( \sum_{n=1}^{NF} |T_n - T_{target}|^2 \right). \tag{15b}
\]
Figure 7a. Sweeping parameters for minimum reflection: the enabled thicknesses and electrical conductivities, and their ranges.

Figure 7b. The optimum values of the parameters after sweeping has been performed.

Figure 8a. The reflection coefficient of the layered medium, with optimum values of the sweeping parameters in Figure 7.

Figure 8b. The electric field distribution within the layered medium at 9 GHz.
Enabling additional parameters as sweeping parameters may lead to even better results. Figure 7a shows the optimization problem configuration where the thicknesses were enabled as sweeping parameters, in addition to the electrical conductivities. After the optimization was performed, the optimum values for both the thicknesses and the electric conductivities were obtained, as shown in Figure 7b. Using these optimum values, the amplitude of the reflection coefficient was plotted in Figure 8a. Comparing Figure 8a to Figure 6 revealed that inclusion of thicknesses in the optimization improved the results significantly. Figure 8b shows the magnitude of the electric field within the layered medium, along with the thickness, using optimum parameters at 9 GHz. The magnitude value on the left boundary, where \( d = 0 \), was very close to one. Since the magnitude of the incident field was one, this indicated that a very low reflection occurred in this medium.

6. Conclusions

An interactive software package was developed to calculate and display the reflection and transmission coefficients of a layered medium. The software can display spatial field distributions as well. The package includes an additional module that performs a parameter sweep to find optimum combinations of parameters for minimum reflection and/or transmission. As future work, the package can be extended to solve for oblique incidence, and the parameter-sweep functionality can be replaced by a global optimizer. This package has been developed and tested using MATLAB 7.1, Release 14, and a p-coded version of this package is available for free download from the Applied Computational Electromagnetics Society (ACES) Web site (http://aces.ee.olemiss.edu) or the authors’ Web site (http://www.ee.olympic.edu/atef).

7. References


2006 Raj Mittra Travel Grant Awards

The 2006 Raj Mittra Travel Grant (RMTG) awards have been announced. The recipient of the Senior Award was Filippo Capolino of the University of Siena, Italy. The recipient of the Junior Award was Mariya Lazebnik of the University of Wisconsin, Madison, WI, USA.

The Raj Mittra Travel Grant Committee received a record number of applicants this year, and with so many excellent candidates, the decisions were very difficult ones, indeed. Our congratulations to the above awardees, and our many thanks to all of those who applied for your interest in the RMTG. The Committee wishes each of you continued success in your research.

Don Wilton, Chair, RMTG Committee
Dept. of Electrical and Computer Engineering
University of Houston
Houston, TX 77204 USA
Tel: +1 (713) 743-4442; Fax: +1 (713) 743-4444
E-mail: wilton@uh.edu